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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 673

VERTICAL DESCENT OF THE AUTOGIRO

By J. A. J. Bennett

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I. SUMMARY

Heretofore it has been tacitly understood that the blade of the autogiro is essentially a windmill. The purpose of this report is to show that only part of the system of rotating blades really is in the "windmill decelerating attitude" when the profile drag is sufficiently low. This particular part receives more torque from the air loads than can be absorbed by the profile drag. As a result thereof the rotating autogiro blade, when its torque is zero, is in part a propeller which functions in the "annular vortex attitude."
(VORTEX RING STATE)

Within this attitude and in the bordering part of "windmill decelerating attitude" the vortex theory of propellers is inapplicable. But Glauert** and Lock *** have defined the characteristic curve in relation to two non-dimensional parameters f and F from the data of propeller experiments in the range where the vortex theory is no longer tenable.

II. INTRODUCTION

In the vortex theory of the propellers it is shown that the field of velocity, defined by the annular ambit of radii r and $r + dr$ and by two planes sufficiently close

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*"Uber den senkrechten Abstieg eines Autogiro." Zeitschrift fur Flugtechnik und Motorluftschiffahrt, April 28, 1932, pp. 219-222.

**Glauert, H.: The Analysis of Experimental Results in the Windmill Brake and Vortex Ring States of an Airscrew. R. & M. No. 1026, British A.R.C., 1926.

***Lock, C. N. H., Bateman, H., and Townend, H. C. H.: An Extension of the Vortex Theory of Airscrews with Applications to Airscrews of Small Pitch, Including Experimental Results. R. & M. No. 1014, British A.R.C., 1926.

before and behind the propeller circle, is perfectly resolved by the angular velocity ω of the propeller, (assuming the rotation in the inflowing slipstream to be zero), and by the mean axial velocity component v_a relative to the propeller circle through the respective circular surface. This approximation is fulfilled quite closely, excepting the immediate vicinity of the tips. These quantities likewise reveal the resultant forces on the wing elements, and for that reason it is also possible to compute the lifting force dH and the torque dM, expressed in ω and v_a from the wind tunnel data for a blade section with respect to infinite aspect ratio for each blade element of radius r.

In order to express the process on a propeller mathematically it is necessary to establish a relationship between quantity v_a at different radii and forward speed v of the propeller, a relation naturally strictly valid in the entire velocity field around the propeller. According to the theory of the vortex fields, the annular particles consisting of air particles which have passed the propeller circle between r and $r + dr$, remain the same even after jet contraction in the eddy zone, where the jet becomes cylindrical again, and the pressure drops to that of the free air. Consequently, v_a/v depends solely on the lift coefficient F relative to v_a:

$$F = (dH/dr)/(4\,\pi\,\rho\,v_a^2\,r).$$

Conformably to the theory on vortex fields the relation between v_a/v and F assumes the simple form of

$$v/v_a\ ?\qquad v_a/v = 1 - F.$$

This relation retains its validity so long as a normal jet development is possible. It is shown to be invalid for sufficiently low v_a (positive or negative). But in this range the relation for the axial inflow (according to the theory of vortex fields) is reestablished by an empiricism, obtained by mathematically expressing the test data on model propellers. (See Lock and Glauert footnotes, page 1.) This empirical relation, shown in Figure 1, interconnects the variables $1/F$ and $1/f$, whereby the dependent variable f is defined as

$$f = (dH/dr)/(4\,\pi\,\rho\,v^2\,r) = (v_a^2/v^2)\ F;$$

thus, f and F are lift coefficients of an element relative to v and v_a.

With $1/f = 0$ (according to fig. 1), the steady condition $v = 0$ is attained, whereas $1/F$ tends toward a finite limiting value which ranges between 1.0 (vortex theory) and 2.0 (Glauert empirical value). When $1/F$ exceeds this limit, v_a and v retain the same prefix; the "normal running attitude" is reached. If $1/F$ drops, v_a and v assume the contrary prefix, the "annular vortex attitude" is obtained up to $1/F = 0$; in this point $v_a = 0$ and $1/f$ assumes a definite limiting value (= 2.0 according to Glauert and Lock).

If $1/f$ exceeds this limit, it results in $v_a/v < 1$, the "windmill decelerating attitude."

III. NOTATION

v_o, speed of descent of autogiro.

w, induced velocity on blade element of length dr.

r, radius of blade element.

R, radius of blade tips.

n, number of blades.

t, chord of blade, assumed constant from root to tip.

α_g, blade angle, also assumed constant, counted from zero lift setting.

α_o, angle of attack of a blade element counted from zero lift setting.

δ, $(\alpha_o - \alpha_g)$

ω, angular velocity of blade.

σ, "fineness" $= \dfrac{\text{blade area}}{\text{blade disk area}} = \dfrac{(n\ t)}{(\pi\ R)}$

ρ, air density (= 0.125 "Newton"/m³ or kg s²/m⁴).

C_a, lift coefficient, with a view to the two-dimensional flow at angle α_0.

C_w, drag coefficient to C_a.

C_{wm}, mean value for C_w between $\alpha_0 = 0$ and $\alpha_0 = \alpha_{0\,max}$ ($\alpha_{0\,max}$ = angle at which the blade is stalled, i.e., about 9° or 0.16 radian for Göttingen wing section No. 429).

H, lifting force along axis of rotor = total weight of autogiro by constant speed of descent.

dH, lifting force, due to the n blade elements of radius r.

dM, torque set up by the n blade elements of radius r.

F and f are defined by

$$dH/dr = 4\,\pi\,r\,\rho\,(v_0 - w)^2\,F \tag{1}$$

$$dH/dr = 4\,\pi\,r\,\rho\,v_0^2\,f \tag{2}$$

x, r/R.

i, blade disk area loading $= H/(\pi\,R^2)$.

c_1, $C_a/(2\,\alpha_0) = 3.0$ (assumed).

x_1, $(v_0/\omega R)^2/(c_1\,\sigma\,\alpha_g/2)$.

β, $(8\sqrt{3}\,\alpha_g)/(c_1\,\sigma)$.

γ, $\beta/(2\,R\,\omega\,\alpha_g) = 4\sqrt{3}/(R\,c_1\,\sigma\,\omega)$

IV. THE TYPICAL CURVE

Glauert's data for the empirical part are:

$1/F$	0	0.25	0.5	0.75	1.0
$1/f$	2.00 2.00	1.08 2.87	0.80 3.17	0.60 3.41	0.50 3.63

The ensuing curve is in fairly close accord with the experimental values, when making allowance for the wind-tunnel effect.

The ambit in which the autogiro blade functions in vertical descent is: $1/F < 1$. The relation between f and F is now expressed by a simple formula. One satisfactory approximation to Glauert's curve is:

$$(1 - 2f)^2 = 3\ f^2/F,$$

but for the purpose at hand

$$(1 - 2f)^2 = 3\ (f/F)^2 \tag{3}$$

is accurate enough. The straights obtained by this (3) are shown in Figure 1.

V. THE BLADE ELEMENT

Disregarding the tangential component of the interference flow, the wind velocity relative to the blade element has the components:

I. ωr perpendicular to the axis of rotation and to the blade element dr.

II. $(v_0 - w)$ perpendicular to the plane of rotation.

The velocity components and the direction of lift and drag, in Figure 2, are for positive δ, that is, when the resultant axial flow through the blade circle is upward. If δ is small, then

$$v_0 - w = \delta\ \omega\ r \tag{4}$$

In the following calculation the stall of the blade toward the root was disregarded as practiced by Glauert*, so that the lift and drag coefficients can be obtained from

$$c_a = 2\, c_1\, \alpha_0 \tag{5}$$

$$c_w = c_{wm} \tag{6}$$

The lifting force and the torque of a blade element are given by

$$dH = n\, t\, \rho\, \omega^2\, r^2\, c_a\, dr/2 \tag{7}$$

$$dM = n\, t\, \rho\, \omega^2\, r^3\, (c_w - \delta\, c_a)\, dr/2 \tag{8}$$

VI. DISTRIBUTION OF INDUCED VELOCITY ALONG THE BLADE

Equations (1) and (2) yield:

$$f/F = (v_0 - w)^2/v_0^2$$

and equation (3)

$$2f = 1 - \sqrt{3}\,(v_0 - w)^2/v_0^2,$$

when $v_0 > w$, that is, when the blade functions in the "windmill decelerating attitude," and

$$2f = 1 + \sqrt{3}\,(w - v_0)^2/v_0^2,$$

when $v_0 < w$, that is, when the blade element functions in the "annular vortex attitude."

Inserted in (2), this attains to

$$dH/dr = 2\,\pi\, r\, \rho\, (v_0^2 - \sqrt{3}\,(v_0 - w)^2) \tag{9}$$

when $v_0 > w$; and

$$dH/dr = 2\,\pi\, r\, \rho\, (v_0^2 + \sqrt{3}\,(w - v_0)^2) \tag{10}$$

when $v_0 < w$.

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*Glauert, H.: A General Theory of the Autogyro. R. & M. No. 1111, British A.R.C., 1926.

Putting $\alpha_0 = \alpha_g + \delta$, (6) and (7) result in

$$dH/dr = c_1\ n\ t\ \rho\ \omega^2\ r^2\ (\alpha_g + \delta) \tag{11}$$

With $\sigma = n\ t/(\pi\ r)$, (9) and (11) yield

$$v_0^2 - \sqrt{3}\ (v_0 - w)^2 = c_1\ \omega\ R\ \sigma\ (\omega\ \alpha_g\ r + v_0 - w)/2,$$

for $v_0 > w$. With

$$\beta = 8\sqrt{3}\ \alpha_g/(c_1\ \sigma),\ \gamma = 4\sqrt{3}/(c_1\ R\ \sigma\ \omega),$$

$$x_1 = (v_0/\omega\ R)^2 \Big/ \left(\frac{c_1\ \sigma\ \alpha_g}{2}\right)$$ and $x = r/R$, we have:

$$\gamma^2\ (v_0 - w)^2 + 2\ \gamma\ (v_0 - w) = \beta\ (x_1 - x),\quad v_0 > w \tag{12}$$

and similarly:

$$\gamma^2\ (w - v_0)^2 + 2\ \gamma\ (w - v_0) = \beta\ (x - x_1),\quad v_0 < w \tag{13}$$

From (12) and (13) follow:

$$\left.\begin{aligned} dw/dx &= \frac{\beta}{2\ \gamma}\ \frac{1}{\gamma\ (v_0 - w) + 1} = \text{positive},\quad v_0 > w \\ dw/dx &= \frac{\beta}{2\ \gamma}\ \frac{1}{\gamma\ (w - v_0) + 1} = \text{positive},\quad v_0 < w \end{aligned}\right\} \tag{14}$$

That is, w increases and $(v_0 - w)$ and δ decrease as x increases along the blade length.

From (12) and (13) follow:

$$\gamma\ (v_0 - w) = -1 + \sqrt{1 + \beta\ (x_1 - x)},\quad v_0 > w \tag{15}$$

$$\gamma\ (w - v_0) = -1 + \sqrt{1 + \beta\ (x - x_1)},\quad v_0 < w \tag{16}$$

The prefix of the roots in both equations was so chosen that (14) is complied with.

The blade element functions in the "windmill decelerating attitude" when $x < x_1$; if $x > x_1$ the "annular vortex state" is reached. When $v_0 = w$, then $x_1 = x$.

In the case of $x_1 < 1$ the streamline pattern is as

in Figure 3. The air particles reverse at the blade tips and describe a closed path about the tip circle. It is to be assumed therefore (see Lock - footnote, page 1) that the zone outside of the blade circle is in a state of turbulence and that the flow intermingles, according to the Tollmien jet mingling theory (Zeitschrift für angewandte Mathematik und Mechanik 6, 1926), conformably to which the momentum transport is explained as motion of the "molecular" particles. In the presence of such turbulence, only that part of the streamline having already passed the circle will return.

VII. ZERO CONDITION FOR THE TORQUE

By uniform motion the torque must become zero. Thus (8) yields

$$\int_0^R n\ t\ \rho\ \omega^2\ r^3\ (C_w - \delta\ C_a)\ dr/2 \tag{17}$$

In the case of $x_1 > 1$, this equation changes because of the insertion of the values for C_w, C_a and δ from (4), (5), and (6), to

$$\frac{C_{wm}\ R^4}{4\ c_1} = \frac{\alpha_g\ R^3}{\omega} \int_0^1 (v_o - w) x^2\ dx + \frac{R^2}{\omega^2} \int_0^1 (v_o - w)^2\ x\ dx \tag{18}$$

Substituting (12) and (15) for $(v_o - w)$ and $(v_o - w)^2$, followed by integration, affords

$$\frac{105\ C_{wm}\ \beta^4}{16\ c_1\ \alpha_g^2} = 105\ \beta^3 \left(\frac{x_1}{2} - \frac{1}{2} + \frac{1}{\beta}\right) - 8(1 + \beta\ x_1)^{5/2}(6 - \beta\ x_1)$$
$$-8(1 - \beta + \beta\ x_1)^{3/2}(\beta^2\ x_1^2 + \frac{3}{2}\beta^2\ x_1 - 5\ \beta\ x_1$$
$$+\frac{15}{8}\beta^2 - 9\ \beta - 6),\quad x_1 > 1 \tag{19}$$

If $x_1 < 1$, equation (17) attains to

$$\frac{C_{wm}\ R^4}{4\ c_1} = \frac{\alpha_g\ R^3}{\omega} \int_0^{x_1} (v_o - w) x^2\ dx - \frac{\alpha_g\ R^3}{\omega^2} \int_{x_1}^1 (w - v_o) x^2\ dx$$
$$+ \frac{R^2}{\omega^2} \int_0^{x_1} (v_o - w)^2\ x\ dx + \frac{R^2}{\omega^2} \int_{x_1}^1 (w - v_o)^2\ x\ dx \tag{20}$$

The replacement of $(v_0 - w)$, $(w - v_0)$, $(v_0 - w)^2$, $(w - v_0)^2$ by equations (15), (16), (12) and (13), respectively, produces when integrated,

$$\frac{105\ \beta^2}{16}\left[\frac{24\ C_{wm}}{c_1^{\,3}\sigma^2} - 2-\beta\ (1 - x_1)\right]$$

$$= \left(6-9\beta - \frac{15}{8}\ \beta^2-5\beta\ x_1 - \frac{3}{2}\ \beta^2\ x_1-\beta^2\ x_1^{\,2}\right)\ (1+\beta-\beta\ x_1)^{3/2}$$

$$+28\ \beta\ x_1 + (\beta\ x_1-6)\ (1+\beta\ x_1)^{5/2},\quad x_1 < 1. \tag{21}$$

VIII. LIFTING FORCE

For $x_1 > 1$, the integration of (9), together with (15) or (12) substituted for $(v_0 - w)$ and $(v_0 - w)^2$ concedes the lifting force. Thus,

$$\frac{\sqrt{3}\ H\ \gamma^2}{2\ \pi\ \rho\ R^2} = (\beta - 3) + \frac{4}{5\ \beta^2}\ [2\ (1 + \beta\ x_1)^{5/2}$$

$$-(2+3\beta+2\beta\ x_1)\ (1-\beta+\beta\ x_1)^{3/2}],\quad x_1 > 1 \tag{22}$$

For $x_1 < 1$, equations (9) and (10) furnish

$$\frac{H}{2\pi\rho R^2} = \int_0^{x_1} [v_0^{\,2} - \sqrt{3}(v_0-w)^2]\ x\ dx + \int_{x_1}^{1} [v_0^{\,2} + \sqrt{3}(w-v_0)^2]\ x\ dx$$

Then the substitution from (12), (13), (15) and (16), followed by integration manifests

$$\frac{\sqrt{3}\ H\ \gamma^2}{2\ \pi\ \rho\ R^2} = \left(\beta + 3 - \frac{16}{15\ \beta^2} - 6\ x_1^{\,2}\right)$$

$$+ \frac{4}{5\ \beta^2}\ [\,2(1+\beta\ x_1)^{5/2} + (2-3\beta-2\beta\ x_1)\ (1+\beta-\beta\ x_1)^{3/2}],$$

$$x_1 < 1 \tag{23}$$

IX. NUMERICAL SOLUTIONS

Equations (19) and (21) afford x_1 as function of the three fundamental quantities α_g, σ and C_{wm}. The definition of these individual values concedes by graphical solution x_1 from either (19) or (21), according to whether $x_1 > 1$ or $x_1 < 1$. The thus obtained values for x_1 are written in (22) or (23), wherefrom the corresponding values for $\omega R \sqrt{(\rho/i)}$ are obtained. (i = loading of blade disk area $(= H/(\pi R^2))$. To make the results more comprehensive, we put $i = 9.76$ kg/m² = 2 lb./sq.ft.

The distribution of the induced velocity can be computed from (15) or (16) and the values for v_0, after defining x_1, from

$$v_0/(\omega R) = (c_1 \sigma \alpha_g x_1/2)^{1/2}.$$

Tables I, II, and III show the effect of the change in fineness, profile drag and blade angle. The results are graphed in Figures 4, 5, and 6.

X. SINKING SPEED

The above calculation affords an explanation for the great parachute action of the autogiro, which means the ratio of drag coefficient (referred to blade disk area: $2 H/(\pi R^2 \rho v_0^2)$) to the drag coefficient of the impermeable annular disk. If this ratio became greater than 1, it was followed by difficulties up to now. In fact, a previous report on propellers with low pitch* drew the conclusion that the drag of a propeller which rotated without torque, cannot exceed the drag of an impermeable annular disk. This conclusion, however, was subsequently computed by experimental tests. (R. & M. No. 1014 - footnote, page 1.) The drag coefficient of an impermeable circular disk in free air is approximately 1.2 and, according to Glauert (R. & M. No. 1111 - footnote, page 6) the maximum value of an autogiro is 2.0. The latter figure rests on the assumption that the mean value of f

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*Lock, C. N. H., and Bateman, H.: Some Experiments on Airscrews at Zero Torque, with Applications to a Helicopter Descending with Engine "Off" and to the Design of Windmills. R. & M. No. 885, British A.R.C., 1923.

over the entire disk area must be lower than 0.5 (see fig. 1), i.e., that on an average the velocity through the disk must be upward.

It is readily apparent that the mean velocity through the disk is mostly zero and that the maximum $(2H/(\pi R^2 \rho v_0^2) = 2.0)$ is practically almost reached, because the evaluation of section IX reveals the value for $2H/(\pi R^2 \rho v_0^2)$ between 1.72 and 1.98; for $\sigma = 0.07$, $\alpha_g = 4^\circ$ and $C_{wm} = 0.01$ it becomes 1.86. Because of the semi-empirical character of this assumption, upon which the present theory was built up, it does not preclude still higher parachute actions, but rather that the limit lies at around 1.7 (i.e., $2H/(\pi R^2 \rho v_0^2) = 2.0$) conformably to the test data obtainable at the present time.

It is perhaps superfluous to add that the steady limiting speed in vertical descent lies above the minimum at which an oblique landing with stopped engine can be effected. The theory of this kind of landing forms the subject of a further investigation.

The writer expresses his appreciation to Senor de la Cierva and to Professor Prandtl for their support in the drafting of this article.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

TABLE I. INFLUENCE OF FINENESS

θ

$\alpha_g = 4^\circ = 0.07$; $C_{wm} = 0.01$

	$\sigma = 0.05$			$\sigma = 0.07$			$\sigma = 0.10$		
β	6.46			4.62			3.23		
x_1	0.98			0.95			0.93		
v_0	9.32			9.20			9.15		
ωR	130			110.2			92.4		
$\gamma\omega R$	46.2			33.0			23.1		
x	(v_0-w)	$10^3\delta$	$10^3\alpha_o$	(v_0-w)	$10^3\delta_o$	$10^3\alpha_o$	(v_0-w)	$10^3\delta$	$10^3\alpha_o$
0.0	4.81	∞	∞	4.42	∞	∞	3.99	∞	∞
0.2	4.11	158	228	3.75	170	240	3.32	180	250
0.4	3.32	64	134	2.94	67	137	2.60	70	140
0.6	2.42	31	101	2.07	31	101	1.76	32	102
0.8	1.32	13	83	1.00	11	81	0.76	10	80
1.0	-0.17	-1	69	-0.37	-3	67	-0.44	-5	65

TABLE II. INFLUENCE OF PROFILE DRAG

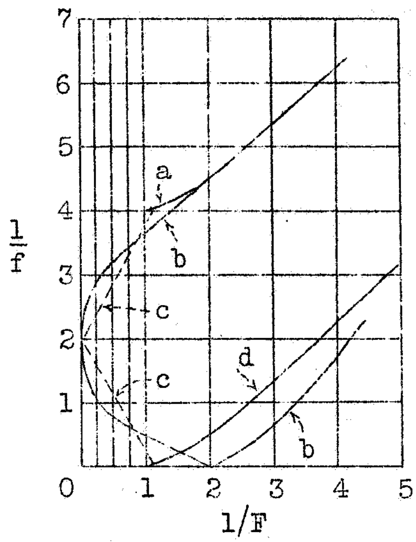
$\alpha_g = 4^\circ \doteq 0.07$; $\sigma = 0.07$

	$C_{wm} = 0$			$C_{wm} = 0.01$	$C_{wm} = 0.02$		
β	4.62				4.62		
x_1	0.69				1.18		
v_0	8.86			See Table I	9.51		
ωR	125				102		
$\gamma\omega R$	33.0				33.0		
x	(v_0-w)	$10^3\delta$	$10^3\alpha_o$		(v_0-w)	$10^3\delta$	$10^3\alpha_o$
0.0	3.96	∞	∞		4.75	∞	∞
0.2	3.05	123	193		4.20	206	276
0.4	2.01	40	110		3.57	87	157
0.6	0.72	10	80		2.84	47	117
0.8	-0.87	-9	61		2.04	25	95
1.0	-2.12	-17	53		1.07	11	81

TABLE III. INFLUENCE OF BLADE ANGLE

$C_{wm} = 0.01$; $\sigma = 0.07$

	$\alpha_g = 0.035$			$\alpha_g = 0.07$	$\alpha_g = 0.105$		
β	2.31			See Table I	6.93		
x_1	1.43				0.84		
v_0	9.54				9.20		
ωR	132				95.5		
$\gamma \omega R$	33.0				33.0		
x	(v_0-w)	$10^3\delta$	$10^3\alpha_0$		(v_0-w)	$10^3\delta$	$10^3\alpha_0$
0.0	4.33	∞	∞		4.66	∞	∞
0.2	3.84	146	181		3.84	202	307
0.4	3.35	64	99		2.93	76	181
0.6	2.84	36	71		1.83	32	137
0.8	2.28	22	57		0.375	5	110
1.0	1.65	12	47		-1.30	-14	91



a, Theory of vortex fields, $f = F/(1 + F)^2$

b, Glauert (empirical)

c, $(1 - 2f)^2 = 3\ (f/F)^2$

d, Theory of vortex fields, $f = F/(1 - F)^2$

Fig. 1

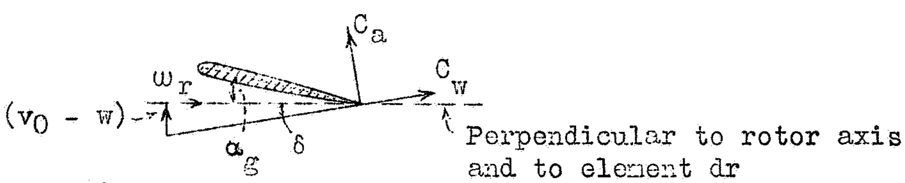


Fig. 2

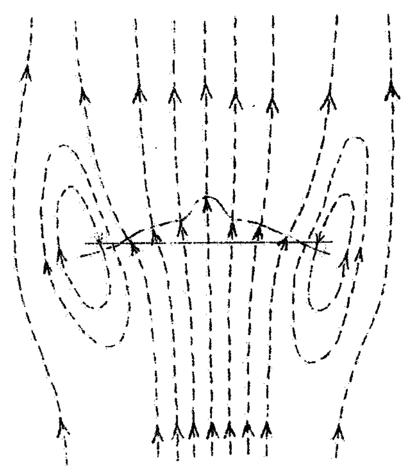
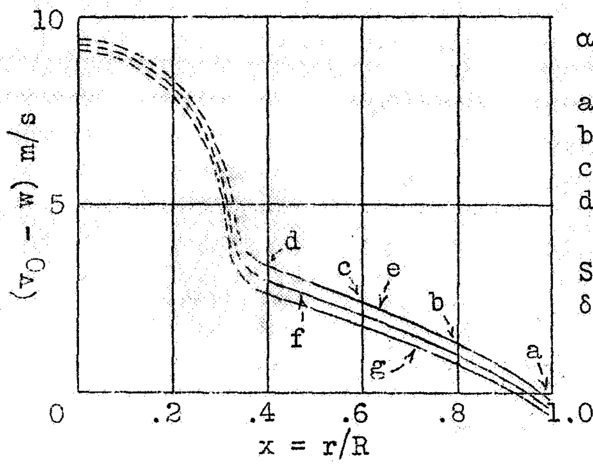


Fig. 3

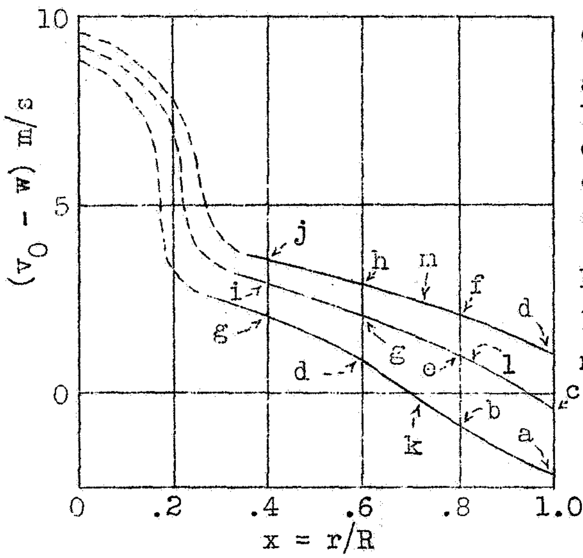


$\alpha_g = 4^\circ$ $C_{wm} = 0.01$

a, $\delta \doteq 4^\circ$ e, $\sigma = 0.05$
b, $\delta \doteq 5^\circ$ f, $\sigma = 0.07$
c, $\delta \doteq 6^\circ$ g, $\sigma = 0.10$
d, $\delta \doteq 8^\circ$

Stalled part of blade -----
$\delta < 9^\circ$ ———

Fig. 4 Influence of fineness



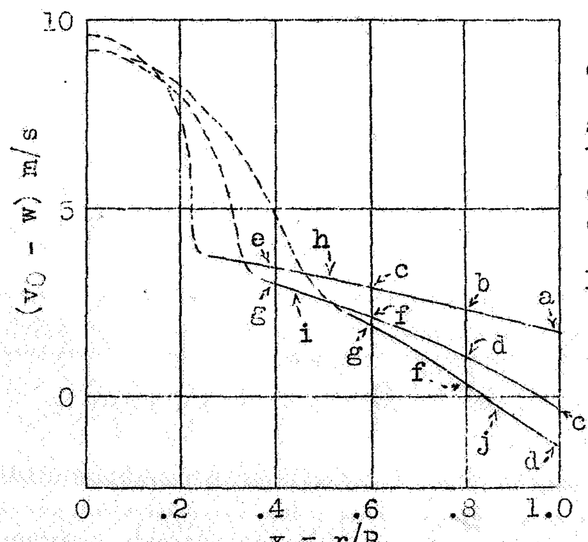
$\sigma = 0.07$ $\alpha_g = 4^\circ$

a, $\delta \doteq 3^\circ$ f, $\delta \doteq 5.5^\circ$
b, $\delta \doteq 3.5^\circ$ g, $\delta \doteq 6^\circ$
c, $\delta \doteq 4^\circ$ h, $\delta \doteq 6.5^\circ$
d, $\delta \doteq 4.5^\circ$ i, $\delta \doteq 8^\circ$
e, $\delta \doteq 5^\circ$ j, $\delta \doteq 9^\circ$

k, $C_{wm} = 0$
l, $C_{wm} = 0.01$
m, $C_{wm} = 0.02$

Stalled part of blade ----
$\delta < 9^\circ$ ———

Fig. 5 Influence of profile drag



$\sigma = 0.07$ $C_{wm} = 0.01$

a, $\delta \doteq 2.5^\circ$ e, $\delta \doteq 5.5^\circ$
b, $\delta \doteq 3.3^\circ$ f, $\delta \doteq 6^\circ$
c, $\delta \doteq 4^\circ$ g, $\delta \doteq 8^\circ$
d, $\delta \doteq 5^\circ$

h, $\alpha_g = 2^\circ$
i, $\alpha_g = 4^\circ$
j, $\alpha_g = 6^\circ$

Stalled part of blade ---
$\delta < 9^\circ$ ———

Fig. 6 Influence of blade angle